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Sensitivity Analysis of Aeroelastic Response of a Wing Using Piecewise Pressure Representation

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A piecewise, panel-based, pressure representation is incorporated to improve the fidelity of an existing wing aeroelastic model, including the sensitivity of the wing static aeroelastic response with respect to various shape parameters. The formulation is quite general and is easily adaptable to any aerodynamics and structural analysis codes desired. An interface code written earlier to transfer information between aerodynamic and structures modules using a global pressure representation has been modified to improve the representation of the calculated pressures using piecewise representations. These improvements are designed to overcome some drawbacks of the earlier representation as well as provide a more accurate representation of the aerodynamic loads to the structures module. A variety of refinements has been attempted to study their effectiveness. A program to combine the local sensitivities, such as the sensitivity of the stiffness matrix or the aerodynamic kernel matrix, into global sensitivity derivatives is developed. Results with excellent accuracy have been produced for large integrated quantities such as wingtip deflection and trim angle of attack. In all cases, accuracy is dependent on the size of the derivatives relative to the size of the quantity itself. The piecewise pressure approaches improved the fidelity of the model and the accuracy of the results at little or no additional computational expense.

Introduction

DURING the design phase of an engineering system, numerous analyses are conducted to predict changes in the characteristics of the system because of changes in design variables. Usually, this process entails perturbing each variable in turn, recalculating the characteristics, and evaluating the sensitivities by a finite difference calculation. These repeated analyses can drive the cost of design very high. An approach that has found increased interest recently in engineering design is analytical calculation of the sensitivity derivatives.[1] Typically, the analytical approach requires less computational resources than a finite difference approach and is less subject to numerical errors (round-off or truncation). The analytical approach is best developed in parallel with the baseline analysis capability since it uses a significant portion of the numerical information generated during that baseline analysis. In the design of modern aircraft, airframe flexibility is a concern from strength, control, and performance standpoints. To properly account for the aerodynamic and structural implications of flexibility, reliable aeroelastic sensitivity analysis is needed. Therefore, both structural and aerodynamic sensitivity analysis capabilities are necessary.

Aeroelastic sensitivity analysis methodology has been available for more than two decades for structural sizing variables.[2] This is because changes in sizing variables exclusively affect the structural stiffness and mass distribution of the airframe and not its basic geometry. Therefore, structural sensitivity analysis capability is sufficient. However, the lack of development in aerodynamic shape sensitivity analysis explains why there are very few results in aeroelastic shape sensitivity analysis.

Barthelemy and Bergen[3] demonstrated the feasibility of calculating analytically the sensitivity of wing static aeroelastic characteristics to changes in wing shape. Kapania et al.[4] obtained the sensitivity of a wing flutter response to changes in the wing's geometry. Specifically, the objective was to determine the derivatives of flutter speed and frequency with respect to wing area, aspect ratio, taper ratio, and sweep angle. The study used Giles' equivalent plate model[5,6] to represent the wing structure. The aerodynamic loads were obtained using Yates' modified strip analysis[7] to analyze flutter characteristics for finite span swept and unswept wings.

A recent paper by Kapania et al.[8] dealt with determining the sensitivity of the various static aeroelastic responses to the variations in various shape parameters, namely: 1) wing area, 2) sweep, 3) aspect ratio, and 4) the taper ratio. The aeroelastic responses were the generalized aeroelastic displacements and the trim angle of attack. The sensitivities were obtained by differentiating the constitutive equations. It was shown that the resulting sensitivity equations can be reformulated into a variation of the Sobieski's global sensitivity equations[9] (G.S.E.) approach. Both schemes gave the various global sensitivities (i.e., the sensitivity including all interdisciplinary interactions) in terms of local sensitivities (i.e., the sensitivities obtained at the discipline level). A key feature that distinguished this study from the study by Barthelemy and Bergen[3] was the use of a more realistic aerodynamic model, FAST,[10] that uses a lifting surface theory as opposed to a lifting line theory employed in the earlier study. The formulation was designed to be quite general so that it was applicable with any aerodynamic code which, for a given geometry and structural deformations, provides aerodynamic pressures on the wing surface. To facilitate the calculation of the shape sensitivities of various quantities (required in aeroelastic analyses), the pressure distribution was

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represented as a double series of global Chebyshev polynomials. The displacements of the wing were obtained using an iterative scheme. To validate this more general formulation, sensitivity of the static aeroelastic response of an example wing was obtained. The results were compared with those obtained by using a purely finite difference approach. A good agreement was obtained.

During the course of the previous work[8] it was found that the generalized pressure coefficients, because of the global nature of the interpolation polynomials, may be sensitive to small changes in independent variables. As a result, the determination of the local derivatives of some of the generalized aerodynamic coefficients was found to be difficult when forward or central differences were used. A higher-order finite difference scheme using a large step size was employed so that the effect of local wiggles can be reduced. Obviously, this was an expensive option. Other more robust techniques are therefore needed to express the aerodynamic pressure. A representation of the aerodynamic loads that is piecewise polynomial (i.e., has a local support) should be used. Some of the advantages of piecewise representation over global representation are discussed by Burden and Faires[11] and de Boor.[12]

In addition, the piecewise polynomial pressure approach will have a major application in representing the aerodynamic pressure of the high-speed civil transport wing. This wing's flow regime will be transonic, thereby having pressure discontinuities from shock waves. A piecewise function is often better represented by piecewise polynomials as opposed to globally defined polynomials.

Sensitivity Equations

A variation of Sobieski's G.S.E. was developed. A variety of local sensitivity data is combined to produce global sensitivity results. Here, the term local sensitivity refers to the sensitivity of an item within a particular discipline, such as the sensitivity of the wing stiffness matrix to a change in wing sweep. Global sensitivities are dependent on the interaction of the disciplines. A global sensitivity example is the variation of wing deflected shape with respect to a change in wing taper ratio.

The governing equations of motion for the aeroelastic analysis and the lift can be written as

$$[K]\{C\} = \{Q\} \tag{1}$$

$$\frac{nW}{2} = \int\int_{\Omega} p(x, y)\, \mathrm{d}\Omega \tag{2}$$

where $[K]$ is the stiffness matrix, $\{C\}$ is the vector of unknown generalized displacements, $\{Q\}$ is the vector of generalized forces, n is the load factor, W is the aircraft weight, $p(x, y)$ is the wing pressure field, and Ω is the one-half wing surface area.

The vector of generalized forces can be obtained as

$$Q_i = \int\int_{\Omega} p(x, y)\gamma_i(x, y)\, \mathrm{d}x\, \mathrm{d}y \tag{3}$$

where $\gamma_i(x, y)$ is the ith nondimensional displacement function used in the displacement model:

$$W(x, y) = \sum_{i=1}^{np} \gamma_i(x, y)C_i \tag{4}$$

These γ satisfy the geometric boundary conditions for a cantilever plate. The C are the generalized displacements.

To facilitate both the integration and subsequent sensitivity calculations, a coordinate transformation was used to simplify the integration limits. Details are given in Ref. 8.

The global Chebyshev pressure representation used in Ref. 8 did not adequately capture small, local, details in the pressure field unless a very large number of terms is used. Thus, an alternative scheme is being employed. The wing is broken into rectangular panels that are remapped to a $(u, v) = (-1, 1)$ square. The pressure is represented by using interpolation polynomials that have local support over the wing panel. It is likely that a larger aerodynamic coefficient vector $\{a\}$ will be necessary for similarly accurate results.

Any number of polynomials are available for this purpose. Three approaches are used in this work: 1) panels with constant pressure, 2) bilinearly interpolated panels, and 3) biquadratically interpolated panels.

The constant panel approach defines a square panel as $(u, v) = (-1, 1)$, and the values at the four corners of the panel as b_{00}, b_{01}, b_{10}, and b_{11}. Then the interpolated value p at any point (u, v) is given by

$$p(u, v) = \frac{1}{4}[1 \quad 1 \quad 1 \quad 1] \begin{Bmatrix} b_{00} \\ b_{01} \\ b_{10} \\ b_{11} \end{Bmatrix} \tag{5}$$

Similarly, the bilinear interpolation panel's pressure is represented as

$$p(u, v) = \frac{1}{4}[(1 - u)(1 - v)(1 - u)(1 + v) \times (1 + u)(1 - v)(1 + u)(1 + v)] \begin{Bmatrix} b_{00} \\ b_{01} \\ b_{10} \\ b_{11} \end{Bmatrix} \tag{6}$$

The form of Eqs. (5) and (6) is suitable for the integration of the forces. If we call the left-hand vector $\boldsymbol{R}$, our interpolation vector, and the right-hand side our aerodynamic coefficients (actually true pressures) $\{a\}$, we get

$$p(u, v) = \{\boldsymbol{R}\}^T\{\boldsymbol{a}\} \tag{7}$$

The biquadratic interpolation scheme uses a nine-node rectangular panel as its basis. The interpolation is performed by the product of two one-dimensional sets of Lagrange polynomials. These polynomials are derived for an arbitrarily positioned inner node:

$$L_1(u) = \frac{1}{2}\frac{(u - u_2)(u - 1)}{(1 + u_2)} \tag{8}$$

$$L_2(u) = \frac{(u + 1)(u - 1)}{(u_2 + 1)(u_2 - 1)} \tag{9}$$

$$L_3(u) = \frac{1}{2}\frac{(u - u_2)(u + 1)}{(1 - u_2)} \tag{10}$$

where u is the location to be interpolated to and u_2 is the coordinate of the middle node.

Then, we can use Eq. (7) if we define our biquadratic interpolation polynomials such as

$$R_1(u, v) = L_1(u)L_2(v) \tag{11}$$

It should also be understood that the biquadratic form of Eq. (7) uses vectors of length nine rather than four.

Generically, the integral for a generalized force Q_i, is

$$Q_i = \int_{-1}^{1}\int_{-1}^{1} p(\eta, \xi)\gamma_i(\eta, \xi)|J_1(\eta, \xi)|\, \mathrm{d}\eta\, \mathrm{d}\xi \tag{12}$$

where $|J_1(\eta, \xi)|$ is the Jacobian of the coordinate transformation. The generalized force Q_i can be written in matrix form as

$$\{Q\} = [A]\{a\} \tag{13}$$

where a typical term in $[A_{LP}]$, our local polynomial aerodynamic matrix, is

$$[A_{LP}] = \sum_{\text{panels}} \int_{-1}^{1} \int_{-1}^{1} R(u, v)\gamma_i(x, y)|J_1(\eta, \xi)||J_2(u, v)| \, du \, dv \tag{14}$$

where $J_2(u, v)$ is the Jacobian of the transformation from the (η, ξ) to (u, v) coordinate system, and the subscript LP indicates a local polynomial interpolation representation of the aerodynamic pressure over the wing panels.

Similarly, the lift equation can be written as

$$\frac{nW}{2} = \sum_{j=1}^{M} a^j L^j = \{L\}^T\{a\} \tag{15}$$

where the $\boldsymbol{L}_{LP}$ vector is very similar to $[A_{LP}]$. Each panel contributes to the four elements of the vector corresponding to its four or nine nodes:

$$\boldsymbol{L}_{LP} = \sum_{\text{panels}} \int_{-1}^{1} \int_{-1}^{1} R(u, v)|J_1(\eta, \xi)||J_2(u, v)| \, du \, dv \tag{16}$$

A Gaussian integration scheme was used to perform the double integrals in the $\boldsymbol{A}_{LP}$ and $\boldsymbol{L}_{LP}$ equations.

The goal of this analysis is to produce values for the global sensitivities dC/dr_l and $d\alpha/dr_l$. Equations (1), (12), and (15) can be used to perform the shape sensitivity analysis of static aeroelastic response. In the following development $\partial(\,)/\partial(\,)$ indicates a local, single discipline, term and $d(\,)/d(\,)$ indicates a global or total derivative. The complete derivation can be found in Ref. 8.

Taking derivatives of the equilibrium and the trim equation, with respect to the shape variable r_l (viz., sweep, aspect ratio, wing area, taper ratio), we obtain

$$\begin{bmatrix} [K] - [A]\left[\dfrac{\partial\{a\}}{\partial\{C\}}\right] & -[A]\left\{\dfrac{\partial\{a\}}{\partial\alpha}\right\} \\ \{L\}^T\left[\dfrac{\partial\{a\}}{\partial\{C\}}\right] & \{L\}^T\left\{\dfrac{\partial\{a\}}{\partial\alpha}\right\} \end{bmatrix} \begin{Bmatrix} \dfrac{d\{C\}}{dr_l} \\ \dfrac{d\alpha}{dr_l} \end{Bmatrix} = \begin{Bmatrix} \left[\dfrac{\partial A}{\partial r_l}\right]\{a\} + [A]\left\{\dfrac{\partial a}{\partial r_l}\right\} - \left[\dfrac{\partial K}{\partial r_l}\right]\{C\} \\ -\left\{\dfrac{\partial L}{\partial r_l}\right\}^T\{a\} - \{L\}^T\left\{\dfrac{\partial a}{\partial r_l}\right\} \end{Bmatrix} \tag{17}$$

The terms $[K]$, $[A]$, $\{a\}$, $\{C\}$, and $\{L\}$ are all quantities known from the converged baseline configuration. A finite difference technique is used to calculate the terms $\partial\{a\}/\partial\alpha$, $\partial[K]/\partial r_l$, and $\partial\{a\}/\partial r_l$. The terms $\partial[A]/\partial r_l$ and $\partial\{L\}/\partial r_l$ are computed analytically. The terms $d\{C\}/dr_l$ and $d\alpha/dr_l$ are the unknowns found by solving this system of equations.

Generating this left-hand sensitivity matrix by finite differences is computationally very expensive, particularly the $\partial\{a\}/\partial\{C\}$ term, which requires calling the aerodynamic code one time for each element in the $\{C\}$ vector. The sensitivity matrix is valid for a particular base geometry, regardless of which r_l is of interest. It is only generated once and saved for future uses.

Implementation

The combination of realistic aerodynamics and structural models in a modular manner with shape sensitivity code requires a systematic approach. A scheme of calling the aerodynamic and structural codes to produce a converged static wing loading and shape was developed. In addition a set of neutral format data files was defined. Here, a neutral format data file is a file that is defined to contain certain data at certain spots, regardless of the package that originally generated the data. This scheme makes the replacement of analysis packages practical and relatively simple. These files include the base geometry and initial deflection values, the intermediate pressure loading and structural deflections, and the final converged wing loading and deflection.

The aeroelastic problem is broken into subproblems (or blocks) by discipline. The aerodynamic and structural blocks are called iteratively to produce a converged static wing loading and shape. Shape sensitivity values for this converged wing are then obtained. These values could then be used in an optimization scheme to modify the baseline geometry.

Each block operates completely independently of the other. It reads from several neutral format input files, performs its calculations, and generates one or more neutral format output files. Thus, any aerodynamic and structural analysis capability may be used. Only new input and output adapter programs need be written to add a new analysis package to the system. These two adapter programs must convert the neutral format data files to and from the new package's native format.

The aerodynamic block is responsible for generating the loads on the wing. It reads as input the wing geometry parameters and the current wing deflections. It is able to output the pressure on the wing at arbitrary points. Currently, the aerodynamic analysis is being performed by the program FAST. This lifting panel code was developed at NASA Langley Research Center. It is based on theory developed by Yates,[7] and was implemented by Desmarais and Bennett.[10]

An evenly distributed rectangular grid was first used. Computer visualization of the error field confirmed that significant error was concentrated in the region very near the leading edge.

A revised gridding scheme was devised. This had two refinements. The inability of FAST to produce pressure data at the wing edges was initially a problem. Under the original local bilinear scheme, panels were assembled away from the edge, resulting in a 100% error band around the outside of the wing. The current scheme puts the panels nearer to the wing edges and also specifies a zero pressure along the edges, allowing panels to be placed along the wing edges.

Second, a variable spacing technique was used to put improved resolution where it was needed. This used a modified half-Gauss–Lobatto grid. The phase and frequency of the cosine term in the Gauss–Lobatto equation was changed to give high resolution at the wing leading edge and tip, and low resolution along the trailing edge and wing root.

The structural block is responsible for calculating the deflection of the wing. It is given the wing geometry and wing loading. It calculates the deflected shape of the wing. Currently, Giles' ELAPS code[5] is being used to perform the structural analysis. This Ritz method program was developed at NASA Langley Research Center. It has been adapted for use on both the VAX and IBM systems. Adapter programs have been developed to convert wing pressures to ELAPS generalized forces and to convert its deflection outputs to neutral form.

The aerodynamic and structural blocks have been combined to produce a converged static wing configuration. The two sections are called iteratively to produce a wing shape and loading that are mutually consistent, for a particular flight condition. This iterative technique is used to keep this model more general. If, in the future, it is desired to use nonlinear aerodynamic or structural codes, it can be done with little additional effort.

Results

This research has produced a variety of useful results. The aeroelastic code does an excellent job of calculating the converged wing loading and deflections for a particular flight condition. The sensitivity derivative calculations do an excellent job of predicting shape sensitivities. Slight difficulty has been encountered in the exact calculation of derivatives with very small values relative to their nominal size. The numerical results were obtained for the wing shown in Fig. 1, at sea level, and Mach 0. It was modeled structurally using a box beam model detailed in Ref. 10. The material properties were $E_{11} = E_{22} = 6.89 \times 10^{10}$, $G_{12} = 2.65 \times 10^{10}$, and $\nu_{12} = 0.3$.

The results from the three local interpolation schemes were nearly identical. With high resolution grids they each converged to nearly the exact same trim angle of attack, tip deflection, and sensitivity values. Thus, only plots for the bilinear approach are included. The variation of the trim angle of attack with respect to the wing area is shown in Fig. 2. The solid line shows the converged results from the iterative aerodynamic and structures combination. The various dashed lines show the variation predicted by the sensitivity derivatives at the different base configurations. The prediction goes through the converged value at the base geometry and is linear with a slope equal to the sensitivity derivative. The desired result is for this line to be tangent to the converged data curve.

Similarly, the sensitivity of the trim angle of attack to changes in the wing aspect ratio is shown in Fig. 3. The solid line shows the converged iterative results and the dashed lines show the predicted variation by having a slope equal to the calculated sensitivity derivative.

Figures 4 and 5 show the converged and predicted values for the angle-of-attack variation with respect to taper ratio and

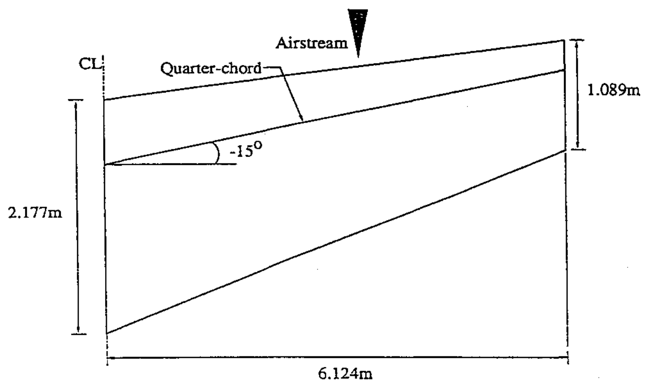

Fig. 1 Baseline wing configuration.

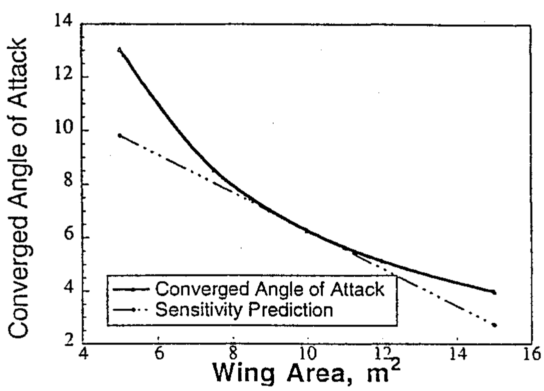

Fig. 2 Trim angle of attack vs wing area.

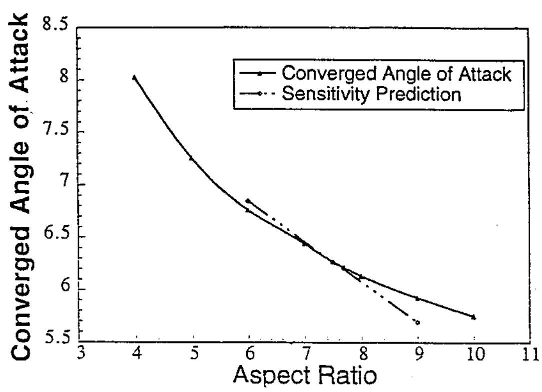

Fig. 3 Trim angle of attack vs wing aspect ratio.

Table 1 Comparison of bilinear and Chebyshev approach derivatives

Term	Bilinear analytic	Chebyshev analytic	Bilinear finite difference
α_{trim}	6.2686	6.0987	——
TD[a]	0.4086	0.3879	——
TD$w/\alpha$	0.6179	0.5915	——
dα/dS	−0.710	−0.689	−0.686
dTD/dS	0.040	0.038	0.040
dα/dAR	−0.766	−0.775	−0.579
dTD/dAR	0.209	0.274	0.209
dα/dΛ	−0.056	−0.051	−0.032
dTD/dΛ	-7.2×10^{-3}	-6.7×10^{-3}	-7.2×10^{-3}
dα/dλ	−0.359	−0.294	−0.188
dTD/dλ	0.213	0.266	0.227

[a]Tip deflection.

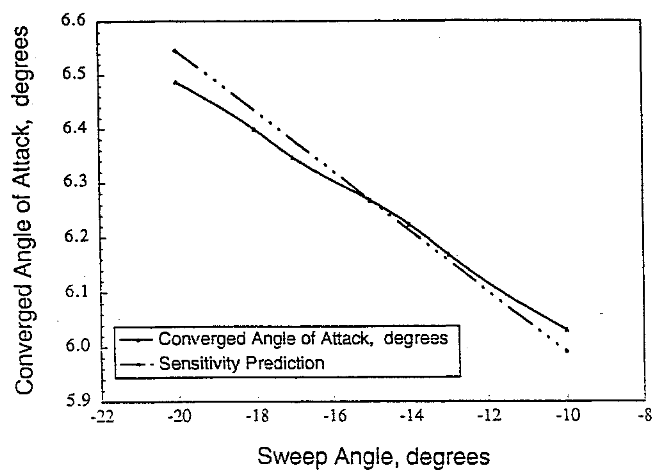

Fig. 4 Trim angle of attack vs wing sweep angle.

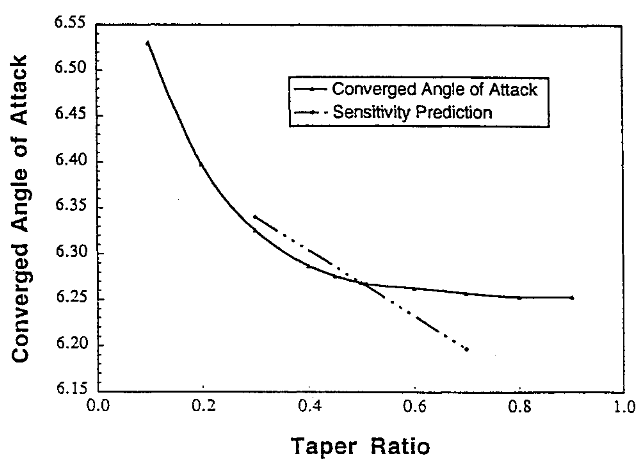

Fig. 5 Trim angle of attack vs wing taper ratio.

sweep. It is obvious from Fig. 5 that the obtained value of the sensitivity of the angle of attack with respect to the taper ratio is not as accurate as one would like. However, note that the value of the converged angle of attack is almost insensitive to the variation in taper ratio at those values of taper ratio. The inaccuracy in the present results can be attributed to the numerical problems associated with determining derivatives that are almost zero.

These few errors are largely numerical in origin. Variables with very small logarithmic derivatives will be difficult to differentiate numerically regardless of the scheme used.[11] The final comparisons between the current versions of both schemes are also of interest. Table 1 compares the various results. The finite difference derivatives are all forward derivatives where the step size was decreased until the derivative converged.

Conclusions

In this research, a variation of Sobieski's global sensitivity equations is implemented to obtain the global sensitivity of the static aeroelastic responses. The scheme is independent of the analysis code used to obtain aerodynamic data.

The results show good accuracy for integrated quantities such as tip displacements, but less accuracy for individual displacement coefficients or trim angle of attack. In general, the accuracy decreases noticeably when the size of the logarithmic derivative decreases.

The global sensitivity approach does an excellent job of predicting global sensitivities with the input of just local sensitivities. It does this without the expense of multiple runs of an entire aeroelastic system.

A few of the capabilities of the current system have never been explored, and could expand the utility of the system. The aerodynamic-code input filter code supports cambered and twisted wings, for instance. The system also allows easy replacement of the various modules with higher performance models. A nonlinear aerodynamic code or a more sophisticated structural module could be easily integrated into the system.

The code implemented for this research can be of significant utility in the early configuration determining stage of a design project. Coupled with an appropriate optimizer, the code could produce a reasonable baseline design for a minimum effort.

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